

TEST A CHAPTER 9, MATHEMATICAL SYSTEMS AND MATRICES

IN PROBLEMS (1-5), FIND THE ANSWER IN CLOCK ARITHMETIC.

_____ 1. $5 + 10 =$

_____ 2. $4 - 10 =$

_____ 3. $4 \times 7 =$

_____ 4. $6 \times 9 =$

_____ 5. $\frac{2}{5} =$

IN PROBLEMS (6-9), $\{0,1,2,3,4\}$ IS THE REPLACEMENT SET FOR n .

_____ 6. Find the value of n if $4 \times 3 \equiv n \pmod{5}$.

_____ 7. Find the value of n if $2 + 3 \equiv n \pmod{5}$.

_____ 8. Find the value of n if $2 - 4 \equiv n \pmod{5}$.

_____ 9. Find the value of n if $\frac{4}{3} \equiv n \pmod{5}$.

IN PROBLEMS (10-11), FIND ALL POSSIBLE REPLACEMENTS FOR n FOR WHICH THE GIVEN CONGRUENCE IS TRUE.

_____ 10. a. $5 + n \equiv 1 \pmod{3}$

_____ b. $6 - n \equiv 4 \pmod{5}$

_____ 11. a. $5n \equiv 1 \pmod{7}$

_____ b. $\frac{n}{4} \equiv 1 \pmod{3}$

THE FOLLOWING TABLE, WHICH DEFINES AN OPERATOR * ON THE SET $S = \{ @, \#, \% \}$, IS USED IN PROBLEMS (12-17).

*	#	@	%
#	%	#	@
@	#	@	%
%	@	%	#

_____12. Is the set S closed under operation $*$? Explain.

_____13. Is the operation $*$ commutative? Explain.

_____14. What is the identity element for the operation $*$?

_____15. For the operation $*$, find the inverse of #.

_____16. For the operation $*$, find the inverse of @.

_____17. For the operation $*$, find the inverse of %.

IN PROBLEMS (18-19), SUPPOSE THAT $a S b$ MEANS TO SELECT THE SECOND OF THE TWO NUMBERS a AND b , AND $a L b$ MEANS TO SELECT THE LARGER OF THE TWO NUMBERS (IF THE NUMBERS ARE EQUAL, SELECT THE NUMBER).

_____18. Is L distributive over S ? Explain.

_____19. Is S distributive over L ? Explain.

THE SET A = {%, @} AND THE OPERATIONS * AND # AS DEFINED IN THE FOLLOWING TABLES WILL BE USED IN PROBLEMS (20-23).

*	%	@
%	%	@
@	@	%

#	%	@
%	%	%
@	%	@

- _____ 20. Find the result of the given operations:
 a. $(\% * @) \# \%$

 b. $(\% \# @) * (\% * @)$.
- _____ 21. Is the set A together with the operation * a group?
- _____ 22. Is the set A together with the operation # a group?
- _____ 23. Does the set A together with the two operations * and # form a field?
- _____ 24. Which of the following payoff matrices is (are) strictly determined?
 (i) $\begin{vmatrix} -2 & 2 \\ 3 & -3 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$ (iii) $\begin{vmatrix} 1 & -3 & 3 \\ -4 & 2 & -2 \\ 2 & 4 & 2 \end{vmatrix}$
- _____ 25. The following payoff matrix is strictly determined. What is the row player's optimum strategy and what is the value of the game?

$$\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$$
- _____ 26. The following payoff matrix is not strictly determined. What fraction of the time should the row player play each row and what is the corresponding payoff value?

$$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

- _____27. Given the three matrices below, find x and y so that $3A - B = C$.

$$A = \begin{vmatrix} 2 & x \\ 4 & y \end{vmatrix}, B = \begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix}, C = \begin{vmatrix} 5 & -2 \\ 8 & 9 \end{vmatrix}$$

- _____28. Calculate AB and BA for the matrices

$$A = \begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}, B = \begin{vmatrix} -1 & 4 \\ 1 & 3 \end{vmatrix}$$

- _____29. In order to build three types of a rolling toy, the Roller Toy Company makes the following demand chart:

TYPE	FRAMES	WHEELS	CHAINS
I	1	2	1
II	1	4	2
III	1	3	2

Suppose that Roller Toy gets an order for 30 type I, 40 type II, and 50 type III of these toys. Use matrices to find the number of frames, wheels, and chains needed to fill this order.

- _____30. Roller Toy finds that frames cost \$3 each, wheels \$1 each, and chains \$2 each. Use matrices to find the total cost of these items for each type of toy in Problem 29.

FOR PROBLEMS (31-34), LET

$$A = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} \quad \text{and} \quad B = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 2 & 0 \\ 2 & 3 & -1 \end{vmatrix}$$

_____ 31. Find $3A - 2B$.

_____ 32. Find $A - B$.

_____ 33. Find $(A - B)^2$.

_____ 34. Find AB and BA .

_____ 35. Use matrices to solve the system:

$$\begin{aligned} x + 2y &= 4 \\ y + 2z &= -1 \\ z + 3x &= 5 \end{aligned}$$

_____ 36. Upon cashing her paycheck for \$475, Polly found that she received all five-, ten-, and twenty-dollar bills. If there were 40 bills in all, and as many twenties as fives, how many of each bill did she get?

- _____37. The augmented matrix of a system of three linear equations in three unknowns x, y, z is reduced to the following form. Find the solution of this system.

$$\left| \begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 1 & 9 \\ 0 & 0 & 2 & 6 \end{array} \right|$$

- _____38. Suppose that the matrix in Problem 37 is changed to read as follows. Find the solutions of this new system.

$$\left| \begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right|$$

- _____39. Suppose that the matrix in Problem 37 is changed to read as follows. What can be said about this system of equations?

$$\left| \begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 1 & 9 \\ 0 & 0 & 0 & 6 \end{array} \right|$$

TEST B CHAPTER 9, MATHEMATICAL SYSTEMS AND MATRICES

1. In clock arithmetic, $5 + 10 =$
a. 1 b. 2 c. 3 d. 4 e. 16

2. In clock arithmetic, $4 - 10 =$
a. 1 b. 3 c. 4 d. 5 e. 6

3. In clock arithmetic, $4 \times 7 =$
a. 2 b. 4 c. 6 d. 8 e. 28

4. In clock arithmetic, $\frac{2}{5} =$
a. 12 b. 5 c. 15 d. 10 e. None of these

5. A value of n that satisfies $2 + 3 \equiv n \pmod{5}$ is
a. 0 b. 1 c. 2 d. 3 e. 4

6. A value of n that satisfies $4 \times 3 \equiv n \pmod{5}$ is
a. 1 b. 2 c. 3 d. 4 e. 5

7. A value of n that satisfies $\frac{4}{3} \equiv n \pmod{5}$ is
a. 1 b. 2 c. 3 d. 4 e. None of these

8. If $5 + n \equiv 1 \pmod{3}$, then all possible values of n are given by
a. $n = 2$ b. $n = 3 + 3k$, k an integer c. $n = -1$
d. $n = 2 + 3k$, k an integer e. None of these

9. If $\frac{n}{2} \equiv 2 \pmod{3}$, then all possible values of n are given by
a. $n = 4$ b. $n = 2$ c. $n = 1$
d. $n = 1 + 3k$, k an integer e. $n = 2 + 3k$, k an integer

THE FOLLOWING TABLE, WHICH DEFINES AN OPERATION * ON THE SET {@, %, \$}, IS USED IN PROBLEMS (10-11).

*	@	%	\$
@	\$	@	%
%	@	%	\$
\$	%	\$	@

10. Which of the following are correct?
- (1) The set S is closed under the operation *.
 - (2) The operation * is commutative.
 - (3) The identity element for * is %.
- a. All three b. (1) and (2) only c. (1) and (3) only
d. (2) and (3) only e. None is correct
11. Which of the following is (are) correct?
- (1) The inverse of @ is %.
 - (2) The inverse of \$ is @.
 - (3) The inverse of % is %.
- a. (1) only b. (2) only c. (3) only
d. (1) and (2) only e. (2) and (3) only

IN PROBLEMS (12-14), a, b, AND c REPRESENT REAL NUMBERS. LET a S b MEAN TO SELECT THE SECOND OF THE TWO NUMBERS a AND b, AND LET a L b MEAN TO SELECT THE LARGER OF THE TWO NUMBERS (IF THE NUMBERS ARE EQUAL, SELECT THE NUMBER).

12. Which of the following is (are) correct?
- (1) The set of real numbers is closed under the operation S.
 - (2) The operation S has the commutative property $a S b = b S a$.
 - (3) The operation S has the associative property $a S (b S c) = (a S b) S c$.
- a. (1) only b. (2) only c. (3) only
d. (1) and (3) only e. (2) and (3) only
13. Which of the following are correct?
- (1) The set of real numbers is closed under the operation L.
 - (2) The operation L has the commutative property $a L b = b L a$.
 - (3) The operation L has the associative property $a L (b L c) = (a L b) L c$.
- a. (1) only b. (1) and (2) only
c. (2) and (3) only d. (1) and (3) only
e. All are correct.

14. Which of the following is (are) correct?
- (1) S is distributive over L , that is, $a S (b L c) = (a S b) L (a S c)$.
 (2) L is distributive over S , that is, $a L (b S c) = (a L b) S (a L c)$.
 (3) For all real numbers a, b, c , $a L (b S c) = a S (b L c)$.
- a. (1) and (2) only b. (1) and (3) only
 c. (2) and (3) only d. (3) only
 e. All three are correct.

THE SET $A = \{\%, @\}$ AND THE OPERATIONS $*$ AND $\#$ AS DEFINED IN THE FOLLOWING TABLES WILL BE USED IN PROBLEMS (15-21).

$*$	$\%$	$@$
$\%$	$\%$	$@$
$@$	$@$	$\%$

$\#$	$\%$	$@$
$\%$	$\%$	$\%$
$@$	$\%$	$@$

15. $(\% * @) \# \% =$
- a. $@ \# @$ b. $@ * \%$ c. $@$ d. $\%$
 e. None of these
16. $(\% \# @) * (\% * @) =$
- a. $@ \# @$ b. $@ * \%$ c. $@$ d. $\%$
 e. None of these
17. Which of the following is (are) correct?
- (1) The set A is closed under $*$.
 (2) $*$ has the associative property.
 (3) The identity element for $*$ is $\%$.
- a. (1) and (2) only b. (1) and (3) only
 c. All three d. (1) only e. (3) only
18. Which of the following is (are) correct?
- (1) $\%$ and $@$ are their own inverses under $*$.
 (2) $*$ does not have the commutative property.
 (3) The set A together with the operation $*$ is a commutative (abelian) group.
- a. (1) and (2) only b. (1) and (3) only
 c. (2) and (3) only d. All three e. (1) only

19. Which of the following is (are) correct?
- (1) The set A is closed under #.
 - (2) # has the associative property.
 - (3) The identity element for # is @.
- a. (1) and (2) only b. (1) and (3) only
 c. All three d. (1) only e. (3) only

20. Which of the following is (are) correct?
- (1) @ is its own inverse, and % has no inverse under #.
 - (2) # has the commutative property.
 - (3) The set A together with the operation # is a commutative (abelian) group.
- a. (1) and (2) only b. (1) and (3) only
 c. (2) and (3) only d. (1) only e. (2) only

21. Which of the following is (are) correct?
- (1) The set A together with the operation * is a group but not a commutative group.
 - (2) The set A together with the operation # is not a group.
 - (3) The set A together with the operations * and # is a field.
- a. (1) and (2) only b. (2) and (3) only
 c. (1) and (3) only d. All three e. (2) only

22. Which of the following payoff matrices is (are) strictly determined?

(i) $\begin{vmatrix} -2 & 2 \\ 3 & -3 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$ (iii) $\begin{vmatrix} 1 & -3 & 3 \\ -4 & 2 & -2 \\ 2 & 4 & 2 \end{vmatrix}$

- a. (i) and (ii) only b. (i) only c. (ii) only
 d. (iii) only e. (ii) and (iii) only

23. The following payoff matrix is strictly determined. What is the row player's optimum strategy and what is the value of the game?

$$\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$$

- a. Play Row 1, value 3 b. Play Row 2, value 2
 c. Play Row 1, value 2 d. Play Row 2, value -1
 e. None of these
24. The following payoff matrix is not strictly determined. What fraction of the time should the row player play each row and what is the corresponding payoff value?

$$\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

- a. Play each row one-half the time, payoff value 2.
 b. Play Row 1 two-thirds of the time and Row 2 one-third of the time, payoff value 2.
 c. Play Row 1 all the time, payoff value 2.
 d. Play Row 1 three-fourths of the time and Row 2 one-fourth of the time, payoff value $1\frac{1}{4}$.
 e. None of these

25. Given the three matrices below, find x and y so that $3A - B = C$.

$$A = \begin{vmatrix} 2 & x \\ 4 & y \end{vmatrix}, B = \begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix}, C = \begin{vmatrix} 5 & -2 \\ 8 & 9 \end{vmatrix}$$

- a. $x = -1, y = -4$ b. $x = 4, y = -1$ c. $x = -1, y = 4$
 c. $x = -4, y = -1$ e. None of these

26. Calculate AB for the matrices $A = \begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}$, $B = \begin{vmatrix} -1 & 4 \\ 1 & 3 \end{vmatrix}$

- a. $\begin{vmatrix} -3 & -16 \\ 1 & -3 \end{vmatrix}$ b. $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ c. $\begin{vmatrix} 0 & -7 \\ -2 & 1 \end{vmatrix}$ d. $\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$

- e. None of these

27. In order to build three types of a rolling toy, the Roller Toy Company makes the following demand chart:

TYPE	FRAMES	WHEELS	CHAINS
I	1	2	1
II	1	4	2
III	1	3	2

Suppose that Roller Toy gets an order for 30 type I, 40 type II, and 50 type III of these toys.

$$\text{If } D = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 3 & 2 \end{vmatrix}, A = [30 \quad 40 \quad 50], B = \begin{vmatrix} 30 \\ 40 \\ 50 \end{vmatrix},$$

then the matrix M giving the numbers of frames, wheels, and chains needed for this order is the product

- a. DA b. AD c. DB
d. BD e. None of these
28. Suppose frames cost \$3 each, wheels \$1 each, and chains \$2 each. Let D be as in Problem 27, let $C = [3 \quad 1 \quad 2]$, and let

$$K = \begin{vmatrix} 3 \\ 1 \\ 2 \end{vmatrix}. \quad \text{The matrix that gives the total cost of these items for each}$$

type of toy is the product

- a. DC b. KD c. CD
d. DK e. None of these

The following two matrices will be used in Problems (29-32).

$$A = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} \quad \text{and} \quad B = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 2 & 0 \\ 2 & 3 & -1 \end{vmatrix}$$

29. $3A - 2B =$

a. $\begin{vmatrix} 5 & 7 & -5 \\ -4 & 0 & 3 \\ 8 & 3 & -7 \end{vmatrix}$ b. $\begin{vmatrix} 5 & 7 & -5 \\ -4 & 2 & 3 \\ -7 & -3 & 8 \end{vmatrix}$ c. $\begin{vmatrix} 5 & -7 & 5 \\ -4 & -2 & -3 \\ -7 & 3 & -8 \end{vmatrix}$

d. $3A - 2B$ is not defined

e. None of these

30. $A - B =$

a. $\begin{vmatrix} -2 & 2 & 2 \\ -2 & 1 & 0 \\ -3 & -2 & 3 \end{vmatrix}$ b. $2 \begin{vmatrix} 2 & -2 & 2 \\ -2 & 0 & 1 \\ -3 & -2 & 3 \end{vmatrix}$ c. $2 \begin{vmatrix} -2 & 2 & 2 \\ 2 & 0 & 1 \\ 3 & -2 & -3 \end{vmatrix}$

d. $A - B$ is not defined

e. None of these

31. $(A - B)^2 =$

a. $\begin{vmatrix} 6 & 8 & 8 \\ 7 & 6 & -7 \\ -11 & -2 & 13 \end{vmatrix}$ b. $-6 \begin{vmatrix} 8 & -8 \\ -6 & -7 \\ -11 & 2 & 13 \end{vmatrix}$ c. $6 \begin{vmatrix} 8 & -8 \\ -6 & 7 \\ -11 & 12 & 13 \end{vmatrix}$

d. $(A - B)^2$ is not defined

e. None of these

32. $BA =$

a. $\begin{vmatrix} -2 & 0 & 4 \\ 2 & 10 & 0 \\ 3 & 11 & -1 \end{vmatrix}$ b. $-2 \begin{vmatrix} 0 & -4 \\ 2 & 0 \\ 3 & 11 & -1 \end{vmatrix}$ c. $-2 \begin{vmatrix} -1 & -4 \\ 2 & 10 & 0 \\ 3 & 11 & 1 \end{vmatrix}$

d. BA is not defined

e. None of these

33. The solution by matrices of the system

$$x + 2y = 4$$

$$y + 2z = -1$$

$$z + 3x = 5$$

gives:

a. $x = -2, y = 1, z = 1$ b. $x = 2, y = 1, z = -1$ c. $x = 2, y = -1, z = 1$

d. $x = 1, y = 2, z = 3$ e. $x = -2, y = -2, z = 1$

34. Upon cashing her paycheck for \$475, Polly found that she received all five-, ten-, and twenty-dollar bills. If there were 40 bills in all, and as many twenties as fives, how many ten-dollar bills did she get?
- a. 5 b. 10 c. 15 d. 8 e. None of these

35. The augmented matrix of a system of three linear equations in three unknowns x, y, z is reduced to the form at the right. The solution of this system is
- $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 3 & 1 & 9 & 0 \\ 0 & 2 & 6 & 0 \end{array} \right]$$

- a. $x = -5, y = -2, z = 3$ b. $x = -5, y = 2, z = 4$
 c. $x = -5, y = 2, z = 3$ d. $x = -5, y = -2, z = 4$
 e. None of these
36. The augmented matrix of a system of three linear equations in three unknowns x, y, z is reduced to the form at the right. If $z = 3k$, where k is any real number, the solution of the system is:
- $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$$
- a. $x = 2 - 7k, y = 3 + k, z = 3k$ b. $x = 2 + 7k, y = 3 - k, z = 3k$
 c. $x = 2 - 7k, y = 3 - k, z = 3k$ d. The system has no solution.
 e. None of these

37. The augmented matrix of a system of three linear equations in three unknowns x, y, z is reduced to the form at the right. This system of equations has
- $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 3 & 1 & 8 \\ 0 & 0 & 0 & 4 \end{array} \right]$$
- a. the solution $z = 0$. b. infinitely many solutions.
 c. the solution $z = k$, k any real number.
 d. the solution $z = 4$. e. no solution.